Chapter 5

The Time Value of Money

5.1 INTRODUCTION

The concept of the time value of money plays a very important role in accounting. According to generally accepted accounting principles (GAAP), assets and liabilities must be presented on the balance sheet at their "present values." Long-term notes receivable and payable, leases, pensions, and amortization of bond premiums and discounts all must take into consideration the value of time. If they do not, they violate GAAP.

What exactly do we mean by the "time value of money" and "present value"? This chapter answers these questions by presenting a detailed description of these concepts together with practical examples.

To begin with, the time value of money involves interest calculations. There are two types of interest: simple interest and compound interest. Under *simple interest*, interest is earned only on the principal; under *compound interest*, interest is earned on the interest as well as on the principal. We will assume the use of compound interest throughout this book, since that is the way it is done in the "real world."

The actual computation of future and present values is done through the use of complex formulas. We will not need to apply these formulas, since interest tables exist that quickly and easily provide the amounts. These tables are presented at the end of this chapter. The tables are:

Table 1. Future Value of $1

Table 2. Present Value of $1

Table 3. Future Value of an Ordinary Annuity of $1

Table 4. Present Value of an Ordinary Annuity of $1

We shall now examine each of these tables and see how they are used.

5.2 THE FUTURE VALUE OF $1

Table 1 answers the following question: "If I deposit $1 today in the bank (or in some other investment), how much will it be worth in the future?" Naturally, it will be worth more than $1 because of the interest factor. But exactly how much will it be worth? Table 1 gives us the answer very quickly. The left-hand column specifies the number of periods involved, while the remaining columns specify the interest rate.

EXAMPLE 1

If I deposit $1 today for 6 years and the interest rate is 5% compounded annually, the table tells us that this will grow into 1.340 ($1.34) in 6 years.

Naturally, nobody makes deposits of just $1. Although this table deals only with $1 deposits, we can easily adapt it for any deposit by simply multiplying the table value by the amount of the deposit.

EXAMPLE 2

If I deposit $1,000 for 10 years and the rate is 10% compounded annually, the table yields a value of 2.594. Multiplying this by the deposit of $1,000 yields a future value of $2,594.

In the previous example, the interest was compounded annually. However, if it is compounded semiannually, we must look up half the rate in the table and double the number of periods. Notice that the table does not use the word *years*; it uses *periods*.

EXAMPLE 3

If I deposit \$1,000 for 10 years and the rate is 10% compounded semiannually, I must look up 5% for 20 periods in the table; the table value is 2.653. Multiplying this by \$1,000 results in \$2,653 for the future value.

If the compounding is quarterly, then we must look up one-fourth the rate in the table and quadruple the number of periods.

EXAMPLE 4

If a deposit of \$1,000 for 10 years at the rate of 12% is compounded quarterly, find the table value at the 3% mark for 40 periods; the table value is 3.262. Multiplying this by \$1,000 yields \$3,262.

Notice from the above examples that the rate is always given on an annual basis. This must then be converted to one-half or one-quarter if the compounding is not on an annual basis.

5.3 THE PRESENT VALUE OF \$1

The present value of \$1 answers the following question: "How much do I have to deposit today to receive \$1 in the future?" This is the opposite side of the coin of the future value of 1. There the question was what will be the amount of the future withdrawal. Here the question is what is the amount of the present deposit.

To answer this question we use Table 2.

EXAMPLE 5

If I wish to withdraw \$8,000 seven years from now and the interest rate is 12% compounded annually, Table 2 yields a value of 0.452; multiplying this by \$8,000 yields \$3,616. This means that to receive \$8,000 seven years from now, I must deposit \$3,616 today.

EXAMPLE 6

Using the same facts as in Example 5 except that the 12% rate is compounded quarterly requires that we look up the table for 28 periods at 3% (12% ÷ 4). The table value is 0.437; multiplying this by \$8,000 yields \$3,496 for today's deposit.

The concepts of the future amount of \$1 and the present amount of \$1 can also be used to answer other questions, such as the number of periods needed to accumulate a certain sum, or what interest rate to invest at.

EXAMPLE 7

If I deposit \$21,545 today, how many years will it take to grow into \$90,000 (with which I intend to purchase a summer cabin), assuming that the rate of interest is 10% compounded annually? This problem can be solved with either Table 1 or Table 2. Let's first use Table 1. We know that \$21,545 multiplied by some table value (which we will call X) should yield \$90,000. Thus,

$$\$21{,}545X = \$90{,}000$$
$$X = 4.17730$$

Looking at Table 1 in the 10% column, we find at the 15-period mark a value that is very close to 4.17730 (4.177). Therefore, the answer is approximately 15 years.

We can also use Table 2 for this problem. In this case, however, the equation has to be set up in reverse:

$$\$90{,}000X = \$21{,}545$$
$$X = 0.23939$$

Looking at Table 2 in the 10% column for 0.23939 yields the same answer of 15 years.

EXAMPLE 8

What rate of interest is needed for a deposit today of $42,241 to grow into $100,000 in 10 years assuming annual compounding?

Using Table 1,

$$\$42{,}241X = \$100{,}00$$
$$X = 2.36737$$

Searching horizontally across Table 1 at the 10-year mark yields a rate of approximately 9%.

5.4 THE FUTURE VALUE OF AN ANNUITY OF $1

The future value of an annuity of $1 answers the following question: "If I make a series of equal deposits of $1 each over several periods, how much will they accumulate to in the future?" Notice the key difference between this situation and the two previous situations. In the previous situation, I made just one deposit; here I am making a series of deposits.

There are two types of annuities: ordinary annuities and annuities due. In the case of an *ordinary annuity*, the deposits are made at the end of each interest period; in the case of an *annuity due*, they are made at the beginning of each interest period. In other words, for ordinary annuities, the deposits begin one period into the future, while for annuities due, they begin immediately today.

EXAMPLE 9

If today is January 1, 19X7, and I plan to make a series of three deposits over the next 3 years with each deposit being made at the end of each year (December 31, 19X7, December 31, 19X8, December 31, 19X9), this is an example of an ordinary annuity. Notice that the deposits do not begin immediately.

However, if each deposit will be made at the beginning of the year (January 1, 19X7, January 1, 19X8, January 1, 19X9), then we are dealing with an annuity due. Notice that the deposits begin right away.

In both cases the withdrawal takes place on December 31, 19X9. In the first case, therefore, the last deposit is made and withdrawn on the same day.

To find the future value of an ordinary annuity, we use Table 3.

EXAMPLE 10

If I make a series of $5,000 deposits at the end of each of the next 5 years and the interest rate is 12% compounded annually, the future value of these deposits will be:

$$\begin{array}{r} 6.353 \\ \times\ \$\ 5{,}000 \\ \hline \$31{,}765 \end{array} \quad \text{(5 deposits, 12\%)}$$

EXAMPLE 11

Assume that I plan to make a series of $1,000 deposits at the end of each 6-month period for the next 5 years and interest is 12% compounded semiannually. In this case, the total number of deposits will be 10 (two per year for 5 years), and each interest period earns 6% (12% × ½).

The table value at 10 deposits and 6% is 13.181. Multiplying this by $1,000 yields a future value of $13,181.

Using some elementary algebra, the table can also be used to calculate the value of each deposit, or the number of deposits that need to be made.

EXAMPLE 12

If I wish to accumulate $20,000 five years from now (in order to make a down payment on a Mercedes) by making a series of deposits at the end of every 3 months, and the interest rate is 12% compounded quarterly, the amount of each deposit can be computed as follows.

The rate per period is 3%, and the number of deposits is 20 (four deposits per year for 5 years). Looking at Table 3 at 20 periods and 3% yields a value of 26.87. Let's call the amount of each deposit X. Then

$$26.870X = \$20,000$$
$$X = \$744.32 \quad \text{(rounded)}$$

Therefore, if \$744.32 is deposited every 3 months for 5 years, the result will be a total of \$20,000.

EXAMPLE 13

Suppose that the future desired value is \$117,332, the annual deposits are \$20,000 each at the end of each year, and interest is compounded annually at 8%. How many deposits have to be made?

Let X = table value. Therefore,

$$\$20,000X = \$117,332$$
$$X = 5.867 \quad \text{(rounded)}$$

Looking at the table in the 8% column for 5.867 yields five deposits.

Table 3 will work only for an ordinary annuity. It cannot be used directly for an annuity due. However, by using a conversion formula, we can adapt the table value and then use it for an annuity due. The formula involves two steps:

Step 1. Look up the table for one additional deposit.

Step 2. Take this value, subtract 1 from it, and then proceed as if you were calculating an ordinary annuity.

EXAMPLE 14

To find the future value of an annuity due of 10 deposits of \$1,000 each, with a 10% rate compounded annually, we would look up the table for 11 deposits and find:

	18.531
Subtract 1	− 1.000
	17.531
Multiply by \$1,000	× 1,000
Yielding a future value of	\$17,531

5.5 THE PRESENT VALUE OF AN ANNUITY OF \$1

The present value of an annuity of \$1 answers the following question: "How much do I have to deposit today to be able to make several equal withdrawals, of \$1 each, over equal periods, in the future?" Note carefully the difference between this and the future value of an annuity. In that situation there were several deposits and one withdrawal. In this situation there is one deposit and several withdrawals.

Once again, we have two types of annuities: ordinary annuities and annuities due. In the former case the first withdrawal is made one period after the deposit, while in the latter case it is made immediately (i.e., right after the deposit is made).

EXAMPLE 15

If today, January 1, 19X7, I made one big deposit with the intention of making three withdrawals starting one period after today (December 31, 19X7, December 31, 19X8, December 31, 19X9), then it is an ordinary annuity situation.

However, if the first withdrawal takes place immediately (i.e., the withdrawal dates are January 1, 19X7, January 1, 19X8, January 1 19X9), then it is an annuity due.